Calculation of the effect of broadband and narrow-band low emissivity coatings on spectral radiant emission and detectability

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Abstract

We present a calculation of the effect of an LPRL narrow-band (band II, 3 – 5 µm) coating and a hypothetical broadband low-emissivity coating (ε = 0.2) on a blackbody heated to 700 K. By restricting the thermal radiation channel the coatings result in an increase in temperature to 771 and 981 K for the LPRL narrow-band coating and the broadband coating, respectively. While both coatings decrease the radiated energy in band II, the LPRL narrow-band coating is more effective; it decreases the energy radiated in band II by 30 %, while the broadband coating results in only a 7 % decrease in energy radiated in band II. These results imply that the maximum detection distance is reduced by 16 % and 4 % for a blackbody coated with the narrow-band coating and a blackbody coated with a broadband coating, respectively, compared to the same blackbody with no coating.

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What happens when a low emissivity coating is applied over an object that is heated by an internal energy source? The temperature at which the object reaches thermal equilibrium with its environment will depend on the emissivity of its surface, all other variables being held constant. For example, consider a cube that is a perfect blackbody radiator. If the cube is heated with an internal source at a given rate, then the cube will come to equilibrium with its environment at a given temperature. Now we imagine applying a low emissivity coating to the cube, and ask at what new temperature will the cube again reach equilibrium with its environment? We suspect that the cube will reach equilibrium at a higher temperature since one channel of thermal exchange (i.e. radiation) with its environment has been diminished.

To calculate the temperature change due to the application of a low emissivity material, we consider a blackbody cube of dimensions of 1 cm per side. Inside the cube is a small but powerful battery that supplies the power needed to heat the cube at any rate needed. With no coating on our blackbody cube, let us heat the cube up to 700 K. The spectral radiant emission that will be given off by the cube is given by Planck's formula :

Equation 1

$$S(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT} - 1}\right),$$

with the output *S* given in Watts per unit volume, and the constants *h*,*c* and *k* have their usual meaning. Using the appropriate conversion factors, the result can be translated into Watts $cm^{-2}\ \mu m^{-1}$, which is what is shown in below in Figure 1 for a temperature of 700 Kelvin.

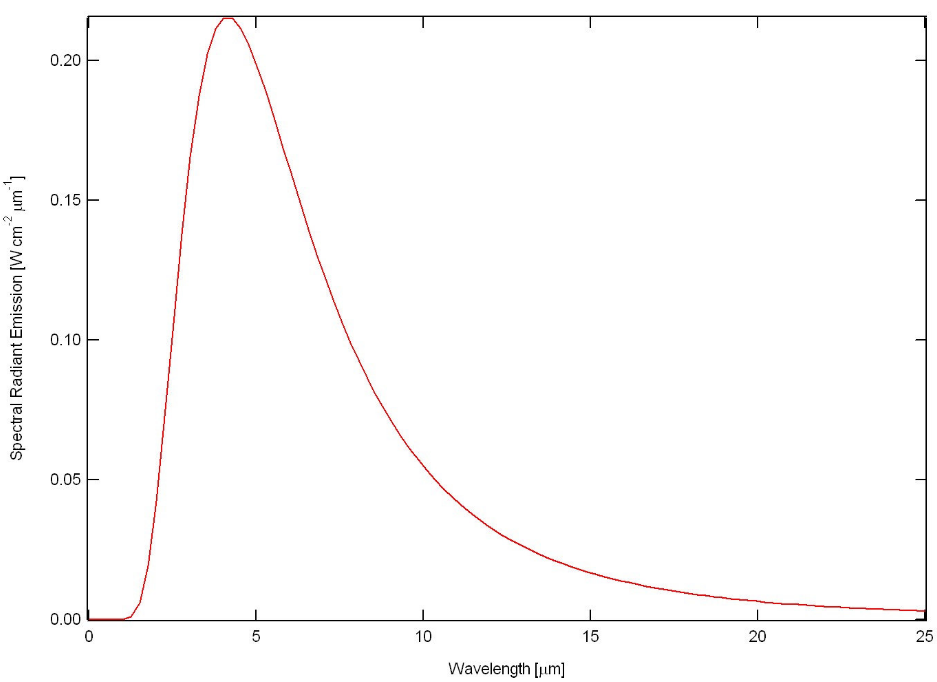

Figure 1: The Spectral Radiant Emission of a blackbody at 700 K.

The total energy flux being radiated by our blackbody cube is the integral over all wavelengths of the spectral radiant emission multiplied by the surface area of the cube (= 6 cm^2).

Equation 2

$$P_{radiated} = 6\int_0^{\infty} \varepsilon(\lambda)S(\lambda,T)d\lambda \text{ [Watts]}$$

The factor $\varepsilon(\lambda)$ is the emissivity of the emitter, which is unity for a black body. Numerically integrating Equation 2 (or using the Stephan-Boltzman law since the emissivity is constant for a blackbody), we find that the power radiated by our blackbody cube due to radiation is approximately 8.2 W.

The energy lost through radiation, however, is less than this since, in addition to radiating energy, the blackbody cube is absorbing energy from the external radiation field. To account for this we first use Kirchhoff's law to equate emissivity and absorptivity. This allows us to assert that the power absorbed will be equal to the power radiated at a given temperature.[1] Hence the total power lost due to radiation is the difference between the radiation emitted by the source at the source temperature and absorbed by the source at the environment's temperature:

Equation 3

$$P_{lost,rad} = 6\int_0^{\infty} \varepsilon(\lambda)\left(S(\lambda,T_{BB}) - S(\lambda,T_{env})\right)d\lambda$$

In Equation 3, T_{BB} and T_{env} are the temperature of the blackbody and of the environment, respectively, and $\varepsilon(\lambda) = 1$ for a black body. Assuming our blackbody cube is radiating into an environment at 20 C, Equation 3 tells us that the power lost by radiation reduces to approximately 7.9 W.

Finally, another channel of by which thermal energy may be lost to the environment is by free convection. This may be estimated using Newton's law of cooling:

Equation 4

$$P_{lost,con} = hA\left(T_{BB} - T_{env}\right)$$

where A is the surface area and h is the convection heat-transfer coefficient. For h we can use a typical value for a machined metallic surface of 10^{-3} Watts cm^{-2} K . The total power lost to the environment can then be estimated as the sum Equations 3 and 4:

Equation 5

$$P_{lost} = P_{lost,rad} + P_{lost,con}$$

[1] Thus at equilibrium in a radiation field at the same temperature as the source, no power is lost to the environment since the power radiated is the same as the power absorbed.

Using Equation 5, we estimate that our black body cube at 700 K loses a total of 10.4 W to the environment through radiation and convection. Our internal heater in the cube must therefore supply 10.4 Watts to maintain the cube at 700 K.

If we now apply a low emissivity coating to all surfaces of the cube without changing the power with which we are heating our cube, the temperature at which the cube will come into thermal equilibrium with its environment can be estimated by demanding that the power lost into the environment, P_{lost}, does not change. The emissivity spectra of the two low emissivity coatings we shall consider are shown in Figure 2.

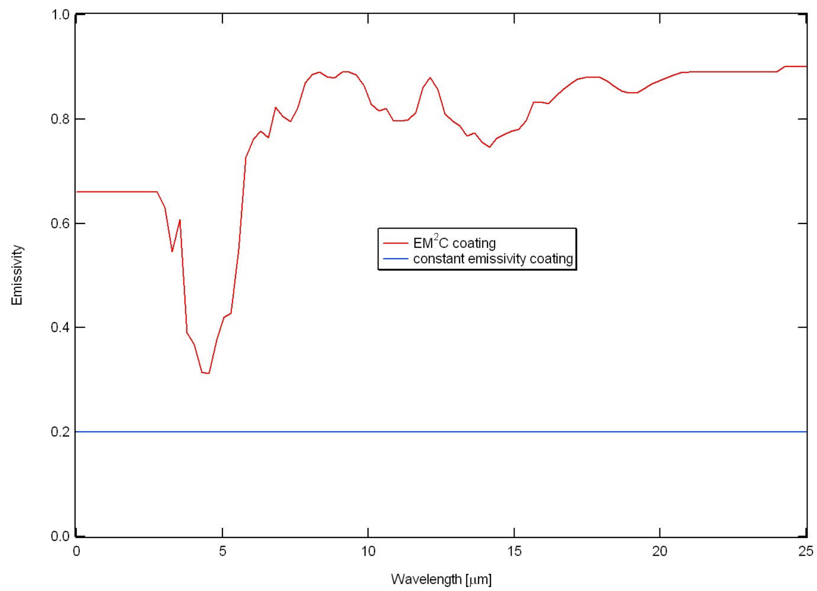

Figure 2: The emissivity of an LPRL coating and a hypothetical coating with constant emissivity of 0.2.

Inserting the two $\varepsilon(\lambda)$ curves from Figure 2 into Equation 3, we can find the temperature at which P_{lost} remains unchanged. The results are given below in Table 1.

Table 1: Equilibrium temperature in Kelvin for blackbody cube, and the same cube coated with indicated low emissivity coatings.

Source	Equilibrium Temperature [Kelvin]
Blackbody (BB)	700 K
CE coated BB	981 K
LPRL coated BB	771 K

Figure 3 shows the spectral radiant emission of the black body cube at equilibrium with its environment with no coating, the LPRL coating, and the constant emissivity (CE) coating. For each of the curves in Figure 3, Equation 5 evaluates to 10.4 Watts.

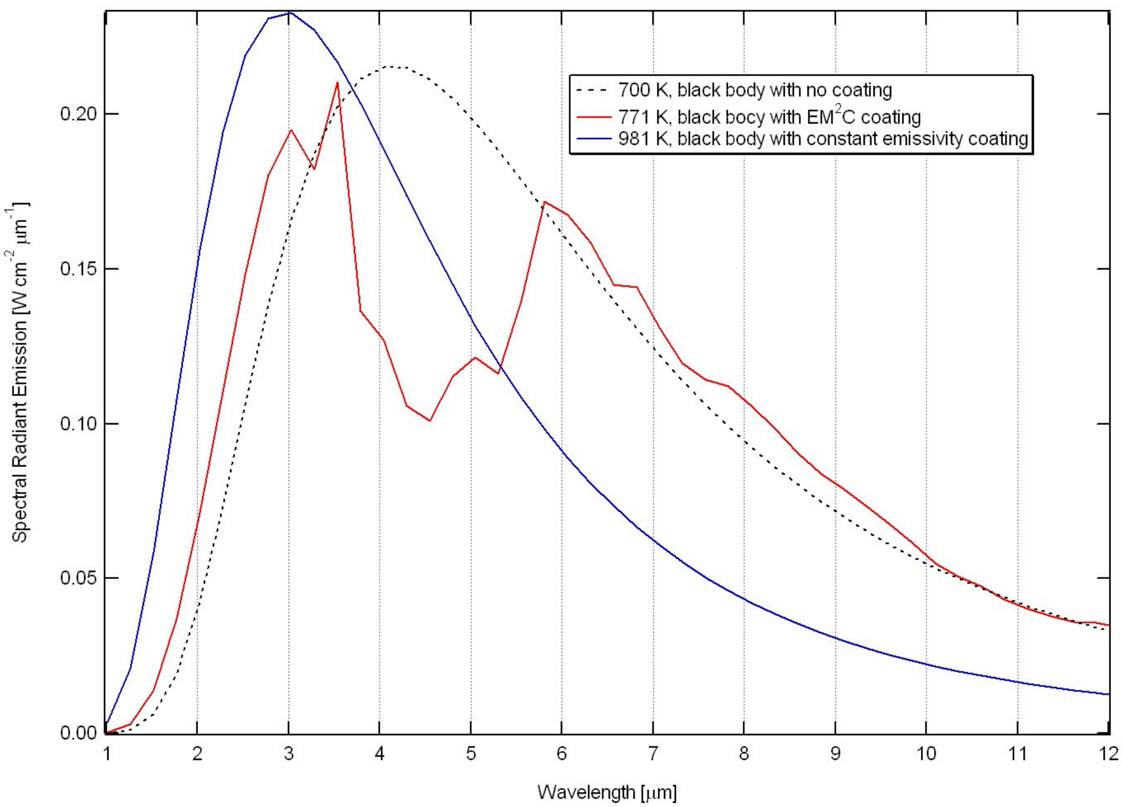

Figure 3: The spectral radiant emission from the blackbocy cube(in equlibrium with its environment) with no coating, and coated with the indicated low emissivity coatings.

We find that the cube coated with the broad-band low emissivity coating results in a much larger increase in temperature of the host than the LPRL coating.

By integrating the curves in Figure 3 from 3 to 5 μm, we can calculate the total power, $P_{radiated}$, emitted in band II, as defined in Equation 2. The result is shown below in Figure 4.

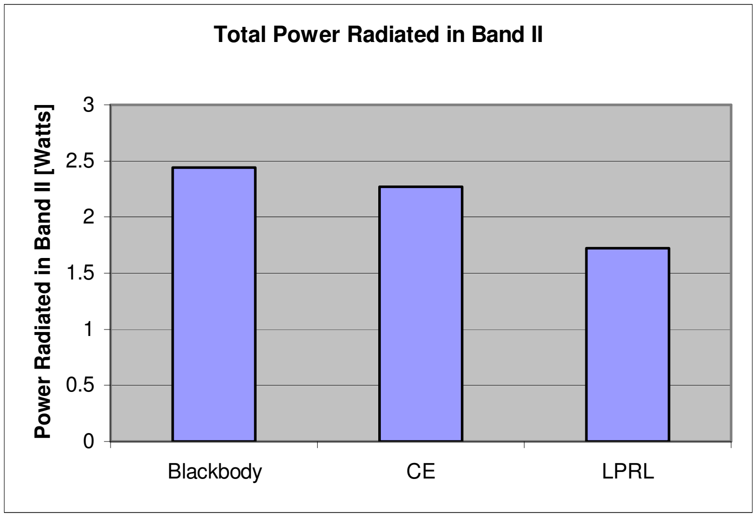

Figure 4: Total power radiated in band II for blackbody, and blackbody coated with indicated coatings. In band II, the blackbody radiates 2.44 W, the CE coated blackbody radiates 2.27 W, and the LPRL coated blackbody radiates 1.72 W.

Both the CE and the LPRL coating lower the power radiated in band II. However, despite having a lower emissivity in band II, the CE coating results in approximately 32% more radiation being emitted in band II as compared to the LPRL coating due to the trapping of thermal energy in the host and the subsequent increase in temperature.

We can now calculate how the lower emissivity affects the detectability of the host. Consider our blackbody cube of 1 cm per dimension heated to 700 K. At a distance >> 1 cm, we can approximate the radiating surface as a point source, so the radiation detected at a distance will decrease as $1/\rho^2$, where ρ is the distance to the source. Thus we can express the power received by a detector that subtends an area A at a distance $\rho >> 1$ cm from the source as

Equation 6

$$P(\rho) = \frac{A}{4\pi} \frac{P_{radiated}}{\rho^2},$$

where $P_{radiated}$ is the total power radiated by the blackbody in band II, as defined in Equation 2. The maximum detection distance will thus scale as the square root of the ratio of power radiated in band II by the emitting source to that of the black body:

Equation 7

$$\rho_{\max} = \rho_{BB} \sqrt{P'/P_{BB}}$$

where ρ_{max} is the maximum distance at which the low-emissivity coated emitter may be detected, ρ_{BB} is the maximum distance at which the blackbody source may be detected, and P' and P_{BB} are the power radiated in band II by the low-emissivity coated emitter and the blackbody, respectively. We thus find that the maximum distance at which the blackbody coated with the CE and LPRL coating may be detected is

Equation 8

$$\rho_{CE} = \rho_{BB}\sqrt{2.27/2.44} = 0.96\rho_{BB};$$
$$\rho_{EM^2C} = \rho_{BB}\sqrt{1.72/2.44} = 0.84\rho_{BB}.$$

We conclude that the maximum distance at which one may detect the blackbody coated with the broadband CE coating is reduced by (1 - 0.96) = 4% compared to the non-coated blackbody. For the blackbody coated with the LPRL coating, the maximum detection distance is decreased by a factor of (1 - 0.84) = 16% compared to the uncoated blackbody.

These results are summarized below in Table 2:

Table 2: Summary of results.

Source	Power Radiated in Band II (arbitrary units)	Maximum Detection Distance[2] (arbitrary units)
Blackbody	1	1
CE coated BB	0.93	0.96
LPRL coated BB	0.73	0.84

[2] Maximum Detection Distance is calculated for a distance >> than the size of the source.